UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M 14 - 180

TITLE	Linear Matrix Equation Solver and General Matrix Inversion
TYPE	Closed routine with two auxiliaries (DOI or SADOI)
NUMBER OF WORDS	115
TEMPORARY STORAGE	0, 1, 2, 3, 4, 5, 6, 7 (also see below)
PRESET PARAMETERS	S3, S4. During input of routine the following parameters are required:

Memory positions $3 = 00\ F\quad 00\ q_1\ F$

$4 = 00\ F\quad 00\ q_2\ F$

where q_1 and q_2 are the first locations of auxiliary routines I and II respectively.

ACCURACY — A function of the order and conditioning of the matrices.

DURATION — n = order of matrix A, m = number of columns of matrix B

$-11 + 4n - 3n^2 + 01n^3 \quad (m = n) \qquad n^3/500 \quad (m = 1)$

DESCRIPTION — Upon appropriate entry, this routine solves the linear matrix equation AX = B or inverts the matrix A, where the matrices A and B must satisfy the conditions:

a) A is non-singular and of size (n x n)

b) B is of size (n x m)

The magnitudes of n and m are governed by the available storage space. In addition to the routine itself, $[\frac{n(n+1)}{2} + n + m + nm]$ memory positions are required directly after the routine. Also, consideration must be given for storing the original matrices A and B and finally the solution X. (See note on Auxiliaries).

ENTRY — A distinction is to be made between the case when B = I (i.e. when we wish to invert A) and the case when $B \neq I$, in which we are in essence solving m sets of n simultaneous equations in n unknowns.

1) B = I: Entry:

p	J0 n
	L5 p
p + 1	26 x
	00 n

2) B ≠ I: Entry:

p	50 n
	L5 p
p + 1	26 x
	00 m

where: n = order of A
m = columns of B
x = first location of this routine.

AUXILIARIES The auxiliaries provide a flexible means for handling the matrices A and B and solution X. By this method one can:

1) successively generate A and B row by row, so that the whole matrix need not be in the machine at one time.
2) have the matrices located on the drum, Williams memory, or on tape.

One can, in the same way, obtain successive columns of X and store them on the drum or punch them on tape so that the complete matrix X need not necessarily be stored in its entirety at any one time.

Auxiliary I: Exit to this subroutine is made from word 22 of the main routine. This auxiliary must provide successive rows of A and B as follows:

a) B = I. The identity matrix is automatically generated in the main routine so that only rows of A must be entered, starting at memory position (x + 115) to (x + 114 + n).
b) B ≠ I. Successive rows of the augmented matrix [A,B] must be provided and read into memory positions starting at (x + 115) to (x + 114 + n + m).

Counting of number of rows (n) is not necessary in the auxiliary nor must provision be made for a link since direct transfer is to be made at the end of the auxiliary to the right hand side of memory position (x + 21).

The matrices must be scaled so that no element is $\geq 1/2$.

Auxiliary II. Since each column of X is computed independently, this auxiliary provides opportunity for disposing of that column before the next one is obtained. Exit to this subroutine is made from word (x + 105); the elements of the respective columns and scaling factor being found in memory position (x + 115) to (x + 115 + n). No counting of number of columns (m) is necessary in the auxiliary, nor must provision be made for a link. Direct transfer is made at the end of the auxiliary to the left hand side of memory position (x + 106).

NOTE Following the n elements of each column of Y is a scaling factor which may be different for each column. It locates the position of the decimal point as lying after the position occupied by the number 1. For example, the sequence

+ 2000	- 0500	+ 2500	+ 1000
- 3000	+ 0050	- 2400	+ 0100
+ 0500	+ 0100	+ 0050	+ 1000

represents the third order matrix

2	-.30	.3
- .5	.3	.1
2.5	-.24	.05

In case we are inverting A the solution $X = A^{-1}$ must be multiplied by 10^{n+1}, if the original matrix A was scaled by 10^{n}.

MATHEMATICAL METHOD See Routine M 13 for mathematical method. The precautions noted there for detecting sinularities of A are incorporated in this routine. Instead of printing out F it will cause a machine hangup from memory position (x + 111) where order ON 3276 NN 3277 is located.

RT: 12/3/58
DATE 5/31/55: RT: 5/29/56
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lgr

LOCATION	ORDER		NOTES PAGE 1
	00 K(M14)		
0	42 15L		Set p
	40 F		Store
1	41 3F		
	41 4F		Clear counters
2	41 7F		
	F5 F		
3	42 10L		Set p + 1
	L4 87L		Right address p + 2
4	46 76L		
	46 83L		
5	42 108L		Link
	10 20F		Set address of y = x + 114 + n
6	42 75L		
	42 91L		
7	42 19L		
	L0 43L		
8	42 3F		Store n
	L5 45L		
9	42 22L		
	L5 91L		Set y
10	42 16L		
	L5 (p+1)F	3	
11	42 4F		Set m
	L4 91L		
12	42 23L		
	42 45L		
13	42 112L		Set address y + m = t
	42 114L		
14	00 20F		
	46 113L		
15	41 6F		Clear counter
	L5 (p)F	0	Test for inversion or solving Ax = B
16	36 21L		
	41 (y)F	10,17'	

LOCATION	ORDER		NOTES PAGE 3
34	46 46L		
	46 65L		
35	42 65L		
	L3 (t)F	24'	Test size of leading elements
36	L6 (x)F	31	
	32 39L		
37	47 45L		No row interchange
	50 7F		Approximately zero in Q
38	L5 (x)F	30'	
	66 (t)F	25	
39	26 43L		
	50 (x)F	29'	
40	33 F		
	32 42L		
41	50 110L		$1 - 2^{-39}$
	75 (t)F	25'	
42	65 (x)F	31'	
	47 46L		Row interchange
43	41 5F		
	51 115L		Address a parameter
44	40 2F		
	L5 (x)F	27,55	
45	40 ( )F	33',37	
	L5 (t)F	12',53	
46	40 ( )F	34,42'	
	50 2F		
47	3J 1F		
	L4 F		
48	40 (x)F	33,54'	
	L3 (x)F	28,54'	
49	L6 5F		Store absolute value of largest element of row
	36 51L		
50	L7 (x)F	32',55	Linearly combine successive rows so as to get zeros
	40 5F		

LOCATION	ORDER		NOTES PAGE 4
51	L5 1F		below the diagonal
	40 (t)F	26,52'	of A
52	F5 51L		
	42 51L		
53	42 45L		
	L5 48L		
54	L4 109L		
	40 48L		
55	46 50L		
	42 44L		
56	L0 113L		
	32 44L		
57	L3 5F		If zero don't rescale
	32 69L		
58	LL 5F		$\geq 1/2$?
	32 61L		
59	L5 66L		
	46 65L		
60	26 64L		If N(5) $\geq 1/2$ multiply row by 1/2 (scale down)
	F5 65L		
61	42 65L		
	L5 5F		
62	00 1F		Determine if possible to scale up
	40 5F		$< 1/2$ N(5) $\leq 1/4$
63	LL 5F		
	32 60L		
64	50 7F		
	L5 (x)F	68,28'	
65	10 (1)F	34',59'	
	00 (1)F	35,61	
66	50 2F		Waste
	40 (x)F	67',29	Rescale rows
67	F5 66L		
	42 66L		
68	42 64L		
	L0 114L		

LOCATION	ORDER		NOTES PAGE 5
69	36 64L		
	F5 6F		
70	40 6F		
	L5 7F		Determine if row i must have further
71	L0 6F		eliminations (done i times)
	36 24L		
72	F5 7F		
	40 7F		
73	L0 3F		Count number of rows
	32 74L		
74	22 8L		Repeat for next row
	41 5F		Set counter
75	L5 111L		
	40 (y)F	6	Set scaling factor
76	L3 (y)F	4	Terminate calculation if scaling $< 2^{-39}$
	36 111L		
77	41 6F		
	41 7F		Clear counters
78	L5 45L		
	42 88L		
79	L0 4F		
	L4 5F		
80	42 83L		
	L5 88L		Set addresses
81	L0 4F		
	42 88L		
82	L5 83L		
	46 88L		
83	50 (y)F	4'	
	71 ( )F	80,86	
84	40 F		
	L5 83L		
85	F0 7F		
	L0 4F		

LOCATION	ORDER		NOTES PAGE 6
86	42 83L		
	22 92L		Calculate $\sum_{j=i+1}^{n} a_{ij}x_j$
87	85 114L		
	50 1F		Also acts as constant
88	50 (y)F	82', 93	
	74 ( )F	78', 81, 93'	
89	L4 F		
	40 F		
90	LL F		
	32 92L		$\geq 1/2$?
91	50 111L		
	7J (y)F	6'	Rescale and start again
92	22 75L		
	L5 88L		
93	L0 109L		
	40 88L		
94	42 98L		Set addresses
	42 101L		
95	46 103L		
	F5 6F		
96	40 6F		
	L5 7F		Count (n-i+1)times for row i
97	L0 6F		
	36 87L		
98	41 6F		Reset counter
	L3 (a_{ii})F	94	End if zero on diagonal
99	36 111L		
	L6 F		
100	36 91L		Test if division is proper
	26 101L		Waste
101	L5 F		
	66 (a_{ii})F	94'	
102	22 102L		Waste
	81 F		

LOCATION	ORDER		NOTES	PAGE 7
103	40 (y-1)F	95		
	F5 7F		Count n rows	
104	40 7F			
	L0 3F			
105	36 S4		Exit to Auxiliary II	
	22 80L		Repeat	
106	F5 5F			
	40 5F		Count m columns	
107	L0 4F			
	32 108L			
108	26 75L		Repeat	
	26 (p+2)F	5	Link	
109	00 1F			
	00 1F			
110	7L 4095F		$1 - 2^{-39}$	
	LL 4095F			
111	00 F			
	00 1000 0000 0000 J		1/10	
112	S6 21L			
	41 (t)F	13		
113	N0 (t)F	14	End constants	
	L3 F			
114	J0 2F			
	40 (t)F	13'		